2.7.4: Direct frequency domain circuit analysis

Overview

In chapter 2.7.2, we determined the steady-state response of electrical circuits to sinusoidal signals using phasor representations of the signals involved, and time-domain representations of the circuit element voltage-current relations. Applying KVL and KCL in this manner resulted in governing equations in which the time dependence had been removed, which converted the governing equations from differential equations to algebraic equations. Unknowns in the resulting algebraic equations were the phasor representations of the signals. These equations could then be solved to determine the desired signals in phasor form; these results could then be used to determine the time-domain representations of the signals.

In chapter 2.7.3, we replaced the time-domain voltage-current relations for passive electrical circuit elements with impedances, which provide voltage-current relations for the circuit elements directly in the frequency domain. At the end of chapter 2.7.3, we used these impedances to schematically represent a circuit directly in the frequency domain.

In this section, we will use this frequency-domain circuit representation to perform circuit analysis directly in the frequency domain using phasor representations of the signals and impedance representations of the circuit elements. This will allow us to write the algebraic equations governing the phasor representation of the circuit directly, without any reference to the time domain behavior of the circuit. As in chapter 2.7.2, these equations can be solved to determine the behavior of the circuit in terms of phasors, and the results transformed to the time domain.

Performing the circuit analysis directly in the frequency domain using impedances to represent the circuit elements results can result in a significant simplification of the analysis. In addition, many circuit analysis techniques which were previously applied to resistive circuits (e.g. circuit reduction, nodal analysis, mesh analysis, superposition, Thevenin’s and Norton’s Theorems) are directly applicable in the frequency domain. Since these analysis techniques have been presented earlier for resistive circuits, in this section we will simply:

1. provide examples of applying these analysis methods to frequency-domain circuits, and
2. note any generalizations relative to using phasors in these analysis methods.

Throughout this section, the student should firmly keep in mind that we are dealing only with the steady-state responses of circuits to sinusoidal forcing functions.
2.7.4: Direct frequency domain circuit analysis

Before beginning this module, you should be able to:

- Use phasors to represent sinusoidal signals (chapter 2.7.1)
- Perform complex arithmetic
- Calculate impedances for resistors, capacitors, and inductors (chapter 2.7.3)

After completing this module, you should be able to:

- State, from memory, KVL and KCL in phasor form
- State, from memory, voltage and current divider formulae in phasor form
- Determine equivalent impedances of parallel and series impedance combinations
- Apply circuit reduction techniques to frequency domain circuits
- Analyze frequency domain circuits using nodal and mesh analysis
- Use superposition to analyze circuits in which multiple frequencies are present
- State Thevenin’s and Norton’s theorems for frequency domain circuits
- Determine the load impedance necessary to deliver maximum power to a load

This module requires:

- N/A
Kirchoff’s Voltage Law:

Kirchoff’s Voltage Law states that the sum of the voltage differences around any closed loop is zero. Therefore, if \( v_1(t), v_2(t), \ldots, v_N(t) \) are the voltages around some closed loop, KVL provides:

\[
\sum_{k=1}^{N} v_k(t) = 0 \tag{1}
\]

Substituting the phasor representation of the voltages results in:

\[
\sum_{k=1}^{N} V_k e^{j\omega t} = 0 \tag{2}
\]

Dividing equation (2) by \( e^{j\omega t} \) results in:

\[
\sum_{k=1}^{N} V_k = 0 \tag{3}
\]

So that KVL states that the sum of the phasor voltages around any closed loop is zero.

Kirchoff’s Current Law:

Kirchoff’s Current Law states that the sum of the currents entering any node is zero. Therefore, if \( i_1(t), i_2(t), \ldots, i_N(t) \) are the currents entering a node, KCL provides:

\[
\sum_{k=1}^{N} i_k(t) = 0 \tag{4}
\]

Substituting the phasor representation of the currents results in:

\[
\sum_{k=1}^{N} I_k e^{j\omega t} = 0 \tag{5}
\]

Dividing equation (2) by \( e^{j\omega t} \) results in:

\[
\sum_{k=1}^{N} I_k = 0 \tag{6}
\]

So that KVL states that the sum of the phasor currents entering (or leaving) a node is zero.

Important result:

KVL and KCL apply directly in the frequency domain.
Example 1: RC circuit steady-state sinusoidal response

In this example, we will revisit example 2 from chapter 2.7.2. In that example, we determined the capacitor voltage in the circuit to the left below, using phasor analysis techniques applied to the circuit’s time-domain governing equation. In this example, we will represent the circuit itself directly in the frequency domain, using impedance representations of the circuit element. The frequency-domain representation of the circuit is shown to the right below.

By the definition of impedance, we can determine the current through the capacitor to be:

\[ I = \frac{Y}{Z_C} = \frac{Y}{1/j \omega C} = j \omega CY \]

The voltage across the resistor can now, by the definition of impedance, be written as \( V_R = R \cdot I = R( j \omega CY ) \). We now apply KVL for phasors to the circuit to the right above, which leads to:

\[ V_p \angle 0^\circ = R( j \omega CY ) + Y \]

Solving for \( Y \) in this equation provides \( Y = \frac{V_p \angle 0^\circ}{1 + j \omega RC} \)

By the rules of complex arithmetic, we can determine the magnitude and phase angle of \( Y \) to be:

\[ |Y| = \frac{V_p}{\sqrt{1 + (\omega RC)^2}} \]
\[ \angle Y = -\tan^{-1}(\omega RC) \]

And the time-domain solution for \( y(t) \) is thus

\[ y(t) = \frac{V_p}{\sqrt{1 + (\omega RC)^2}} \cos\left[\omega t - \tan^{-1}(\omega RC)\right] \]
Parallel and Series Impedances & Circuit Reduction

Consider the case of N impedances connected in series, as shown in Figure 1. Since the elements are in series, and since we have seen that KCL applies to phasors, the phasor current $I$ flows through each of the impedances. Applying KVL for phasors around the single loop, and incorporating the definition of impedance, we obtain:

$$V = I( Z_1 + Z_2 + \cdots + Z_N ) = 0$$  \hspace{3cm} (7)

Figure 1. Series combination of impedances.

If we define $Z_{eq}$ as the equivalent impedance of the series combination, we have $V = I \cdot Z_{eq}$, where

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_N$$  \hspace{3cm} (8)

so that impedances in series sum directly. Thus, impedances in series can be combined in the same way as resistances in series.

By extension of the above result, we can develop a voltage divider formula for phasors. Without derivation, we state that the phasor voltage across the $k^{th}$ impedance in a series combination of N impedances as shown in Figure 1 can be determined as:

$$V_k = V \frac{Z_k}{Z_1 + Z_2 + \cdots + Z_N}$$  \hspace{3cm} (9)

so that our voltage division relationships for resistors in series apply directly in the frequency domain for impedances in series.

We now consider the case of N impedances connected in parallel, as shown in Figure 2. Since the elements are in parallel, and KVL applied to each loop shows that all circuit elements share the same phasor voltage difference $V$. Applying KCL for phasors at the upper node, and incorporating the definition of admittance, we obtain:

$$I = V( Y_1 + Y_2 + \cdots + Y_N ) = 0$$  \hspace{3cm} (10)
If we define $Y_{eq}$ as the equivalent impedance of the series combination, we have:

$$I = V \cdot Y_{eq}$$  \hspace{1cm} (11)$$

where

$$Y_{eq} = Y_1 + Y_2 + \cdots + Y_N$$  \hspace{1cm} (12)$$

so that admittances in series sum directly. Converting our admittances to impedances indicates that the equivalent impedance of a parallel combination of N impedances as shown in Figure 2 is:

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}}$$  \hspace{1cm} (13)$$

Thus, impedances in parallel can be combined in the same way as resistances in parallel.

By extension of the above result, we can develop a current divider formula for phasors. Without derivation, we state that the phasor current across the k\textsuperscript{th} impedance in a series combination of N impedances as shown in Figure 1 can be determined as:

$$I_k = I \frac{1/}{Z_k} \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}}$$  \hspace{1cm} (14)$$

so that our current division relationships for resistors in parallel apply directly in the frequency domain for impedances in parallel.

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**Important result:**

All circuit reduction techniques for resistances apply directly to the frequency domain for impedances. Likewise, voltage and current divider relationships apply to phasor circuits in the frequency domain exactly as they apply to resistive circuits in the time domain.
Example 2: Use circuit reduction techniques to determine the current phasor $I$ leaving the source in the circuit below. (Note: the circuit below is the frequency domain circuit we obtained in example 1 of chapter 2.7.3.)

Since impedances in series add directly, the inductor and resistor can be combined into a single equivalent impedance of $(2+j1)\Omega$, as shown in the figure to the left below. The capacitor is then in parallel with this equivalent impedance. Since impedances in parallel add in the same way as resistors in parallel, the equivalent impedance of this parallel combination can be calculated by dividing the product of the impedances by their sum, so

$$Z_{eq} = \frac{(-j3)(2+j1)}{(-j3)+(2+j1)} \Omega = \frac{3-j6}{2-j2} \Omega.$$  

Converting this impedance to polar form results in $Z_{eq} = 2.37 \angle -18^\circ \Omega$; the final reduced circuit is shown in the figure to the right below.

Using the reduced circuit to the right above and the definition of impedance, we can see that:

$$I = \frac{20 \angle 30^\circ}{2.37 \angle -18^\circ \Omega} = \frac{20}{2.37} \angle [30^\circ - (-18^\circ)] A$$  

so that

$$I = 8.44 \angle 48^\circ A$$
Example 3: Use circuit reduction techniques to determine the current, $i(t)$ through the inductor in the circuit below.

With $\omega = 2 \text{ rad/sec}$, the frequency domain representation of the circuit is as shown in the figure to the left below; in that figure, we have also defined the current phasor $I_S$ leaving the source.

We now employ circuit reduction techniques to determine the phasor $I$. To do this, we first determine the circuit impedance seen by the source; this impedance allows us to determine the source current $I_S$. The current $I$ can be determined from a current divider relation and $I_S$.

The impedances of the series combination of the capacitor and the $4\Omega$ resistor is readily obtained by adding their individual impedances, as shown in the figure to the left below. This equivalent impedance is then in parallel with the inductor's impedance; the equivalent impedance of this parallel combination is as shown in the circuit to the right below.

The source current is then, by the definition of impedance, $I_S = \frac{5 \angle 0^\circ}{4 - j4\Omega + 2\Omega} = 0.69 \angle -33.7^\circ$.

The circuit to the left above, along with our voltage divider formula, provides:

$$I = \frac{(4 - j4\Omega)}{4 - j4\Omega + j4\Omega} \cdot I_S = (1 - j1\Omega) \cdot 0.69 \angle -33.7^\circ = 0.98 \angle -78.7^\circ$$

And the current $i(t) = 0.98 \cos(2t - 78.7^\circ)$
Nodal analysis and mesh analysis techniques have been previously applied to resistive circuits in the time domain. In nodal analysis, we applied KCL at independent nodes and used Ohm’s Law to write the resulting equations in terms of the node voltages. In mesh analysis, we applied KVL and used Ohm’s Law to write the resulting equations in terms of the mesh currents.

In the frequency domain, as we have seen in previous sub-sections, KVL and KCL apply directly to the phasor representations of voltages and currents. Also, in the frequency domain, impedances can be used to represent voltage-current relations for circuit elements in the frequency domain in the same way that Ohm’s Law applied to resistors in the time domain (the relation \( V = I \cdot Z \) in the frequency domain corresponds exactly to the relation \( v(t) = R \cdot i(t) \) in the time domain). Thus, nodal analysis and mesh analysis apply to frequency domain circuits in exactly the same way as to time domain resistive circuits, with the following modifications:

- The circuit excitations and responses are represented by phasors.
- Phasor representations of node voltages and mesh currents are used.
- Impedances are used in the place of resistances.

Application of nodal and mesh analysis to frequency-domain circuit analysis is illustrated in the following examples.

**Example 4: Use nodal analysis to determine the current \( i(t) \) in the circuit of example 3.**

The desired frequency-domain circuit was previously determined in Example 3. Nodal analysis of the frequency-domain circuit proceeds exactly as was done in the case of resistive circuits. The reference voltage, \( V_R = 0 \), and our single node voltage, \( V_A \), for this circuit are defined on the circuit below.

![Circuit Diagram](image)

Applying KCL in phasor form at node A provides:

\[
\frac{5\angle 0^\circ - V_A}{2\Omega} - \frac{V_A}{(4 - j4)\Omega} - \frac{V_A}{j4\Omega} = 0
\]

Solving for \( V_A \) gives \( V_A = 3.92\angle 11.31^\circ V \). By the definition of impedance, the desired current phasor \( I = \frac{V_A}{j4\Omega} = \frac{3.92\angle 11.31^\circ}{4\angle 90^\circ} = 0.98\angle -78.7^\circ \) so that \( i(t) = 0.98\cos(2t - 78.7^\circ) \), which is consistent with our result obtained via circuit reduction in Example 3.

**Example 5: Use mesh analysis to determine the current \( i(t) \) in the circuit of examples 3 and 4.**
The desired frequency-domain circuit was previously determined in Example 3. Mesh analysis of the frequency-domain circuit proceeds exactly as for resistive circuits. The figure below shows our choice of mesh loops; the series resistor-capacitor combination has been combined into a single equivalent resistance in the figure below, for clarity.

\[
\begin{align*}
5 \angle 0^\circ &= (4-j4)(I_1 - I_2) = 2\Omega
\end{align*}
\]

KVL around the mesh loop \( I_1 \) provides:

\[
5 \angle 0^\circ - 2 \cdot I_1 - (4-j4)(I_1 - I_2) = 0
\]

KVL around the mesh loop \( I_2 \) provides:

\[
(4-j4)(I_2 - I_1) + j4 \cdot I_2 = 0
\]

The second equation above can be simplified to provide: \( I_2 = (1 - j)I_1 \). Using this result to eliminate \( I_1 \) in the mesh equation for loop \( I_1 \) and simplifying provides:

\[
5 \angle 0^\circ = \left[ \frac{(6-j4)}{1-j} + (j4-4) \right] I_2
\]

so that \( I_2 = 0.98 \angle -78.7^\circ \). The mesh current \( I_2 \) is simply the desired current \( I \), so in the time domain,

\[
i(t) = 0.98 \cos(2t - 78.7^\circ)
\]

which is consistent with our results from examples 3 and 4.

**Important result:**

Nodal and mesh analysis methods apply to phasor circuits exactly as they apply to resistive circuits in the time domain. Impedances simply replace resistances, and quantities of interest become complex valued.
Superposition:

The extension of superposition to the frequency domain is an extremely important topic. Several common analysis techniques you will encounter later in this course and in future courses (frequency response, Fourier Series, and Fourier Transforms, for example) will depend heavily upon the superposition of sinusoidal signals. In this sub-section, we introduce the basic concepts involved.

In all of our steady-state sinusoidal analyses, we have required that the circuit is linear. (The statement that the steady state response to a sinusoidal input is a sinusoid at the same frequency requires the system to be linear. Nonlinear systems do not necessarily have this characteristic.) Thus, all phasor circuits are linear and superposition must apply. Thus, if a phasor circuit has multiple inputs, we can calculate the response of the circuit to each input individually and sum the results to obtain the overall response. It is important to realize, however, that the final step of summing the individual contributions to obtain the overall response can, in general, only be done in the time domain. Since the phasor representation of the circuit response implicitly assumes a particular frequency, the phasor representations cannot be summed directly. The time domain circuit response, however, explicitly provides frequency information, allowing those responses to be summed.

In fact, because the frequency-domain representation of the circuit depends upon the frequency of the input (in general, the impedances will be a function of frequency), the frequency domain representation of the circuit itself is, in general, different for different inputs. Thus, the only way in which circuits with multiple inputs at different frequencies can be analyzed in the frequency domain is with superposition.

In the special case in which all inputs share a common frequency, the circuit response can be determined by any of our previous analysis techniques (circuit reduction, nodal analysis, mesh analysis, superposition, etc.) In this case, if superposition is used, the circuit response to individual inputs can be summed directly in the frequency domain if desired.

Examples of the application of superposition to analysis of frequency-domain circuits are provided below.

Important result:

In the case of multiple frequencies existing in the circuit, superposition is the only valid frequency-domain analysis approach.

Superposition applies directly in the frequency domain, insofar as contributions from individual sources can be determined by killing all other sources and analyzing the resulting circuit. In general, however, superimposing (summing) the contributions from the individual sources must be done in the time domain.

Superposition of responses to individual sources can be summed directly in the frequency domain (e.g. addition of the phasors representing the individual responses) is only appropriate if all sources have the same frequency. In this case (all source having the same frequency) any of our other modeling approaches are also valid.
Example 6: Determine the voltage $v(t)$ across the inductor in the circuit below.

Since two different input frequencies are applied to the circuit, we must use superposition to determine the response. The circuit to the left below will provide the phasor response $V_1$ to the current source: the frequency is $\omega = 9 \text{ rad/sec}$ and the voltage source is killed. The circuit to the right below will provide the phasor response $V_2$ to the voltage source: the frequency is $\omega = 3 \text{ rad/sec}$ and the current source is killed.

To determine the voltage phasor resulting from the current source ($V_1$ in the circuit to the left above), we note that the inductor and the $3\Omega$ resistor form a current divider. Thus, the current through the inductor resulting from the current source is

$$I_1 = \frac{3\Omega}{(3 + j3)\Omega} \cdot 6\angle 0^\circ = \frac{3\Omega \cdot 6\angle 0^\circ}{3\sqrt{2}\angle 45^\circ} = \frac{6}{\sqrt{2}} \angle -45^\circ.$$

The voltage phasor $V_1$ can then be determined by multiplying this current times the inductor’s impedance:

$$V_1 = j3\Omega \cdot \frac{6}{\sqrt{2}} \angle -45^\circ = 3\angle 90^\circ \cdot \frac{6}{\sqrt{2}} \angle -45^\circ = 9\sqrt{2}\angle 45^\circ V$$

and the time-domain voltage across the inductor due to the current source is:

$$v_1(t) = 9\sqrt{2} \cos(9t + 45^\circ) V$$

To determine the voltage phasor resulting from the voltage source ($V_2$ in the circuit to the right above), we note that the inductor and the $3\Omega$ resistor now form a voltage divider. Thus, the voltage $V_2$ can be readily determined by:

$$V_2 = \frac{j1\Omega}{(3 + j1)\Omega} \cdot 4\angle 30^\circ = \frac{1\angle 90^\circ \cdot 4\angle 30^\circ}{\sqrt{10}\angle 18.4^\circ} = \frac{4}{\sqrt{10}} \angle 101.6^\circ$$
So that the time-domain voltage across the inductor due to the voltage source is:

\[ v_2(t) = \frac{4}{\sqrt{10}} \cos(3t + 101.6^\circ) \text{V} \]

The overall voltage is then the sum of the contributions from the two sources, in the time domain, so:

\[ v(t) = v_1(t) + v_2(t) \]

And

\[ v(t) = 9\sqrt{2} \cos(9t + 45^\circ) + \frac{4}{\sqrt{10}} \cos(3t + 101.6^\circ) \text{V} \]

Example 7: Determine the voltage \( v(t) \) across the inductor in the circuit below.

This circuit is essentially the same as the circuit of Example 6, with the important difference that the frequency of the voltage input has changed – the voltage source and current source both provide the same frequency input to the circuit, 9 rad/sec. We will first do this problem using superposition techniques. We will then use nodal analysis to solve the problem, to illustrate that multiple inputs at the same frequency do not require the use of superposition.

Individually killing each source in the circuit above results in the two circuits shown below. Note that the impedance of the inductor is now the same in both of these circuits.

The two circuits shown above will now be analyzed to determine the individual contributions to the inductor voltage; these results will then be summed to determine the overall inductor voltage.
The circuit to the left above has been analyzed in Example 6. Therefore, the voltage phasor $V_1$ is the same as determined in Example 6:

$$V_1 = 9\sqrt{2}\angle 45^\circ V$$

The voltage $V_2$ in the circuit to the right above can be determined from application of the voltage divider formula for phasors:

$$V_2 = \frac{j3\Omega}{(3 + j3)\Omega} \cdot 4\angle 30^\circ = \frac{3\angle 90^\circ \cdot 4\angle 30^\circ}{3\sqrt{2}\angle 45^\circ} = 2\sqrt{2}\angle 75^\circ$$

Since both inputs have the same frequency, we can superimpose the phasor results directly (we could, of course, also determine the individual time domain responses and superimpose those responses if we chose):

$$V = V_1 + V_2 = 9\sqrt{2}\angle 45^\circ + 2\sqrt{2}\angle 75^\circ = 15.24\angle 50.3^\circ V$$

So that the time domain inductor voltage is $v(t) = 15.24\cos(9t + 50.3^\circ )V$. Notice that the circuit response has only a single frequency component, since both inputs have the same frequency.

The superposition approach provided above is entirely valid. However, since both sources have the same input, we can choose any of our other analysis approaches to perform this problem. To emphasize this fact, we choose to do this problem using nodal analysis.

The frequency-domain circuit, with our definition of reference voltage and independent node, is shown in the figure below.

KCL at node A provides:

$$6\angle 0^\circ = \frac{V_A - 0}{j3\Omega} + \frac{V_A - 4\angle 30^\circ}{3\Omega}$$

Solving the above equation for $V_A$ provides $V_A = 15.24\angle 50.3V$ so that the inductor voltage as a function of time is:

$$v(t) = 15.24\cos(9t + 50.3^\circ )V$$

Which is consistent with our result using superposition.
Application of Thévenin’s and Norton’s Theorems to frequency domain circuits is identical to their application to time domain resistive circuits. The only differences are:

- the open circuit voltage ($v_{oc}$) and short circuit current ($i_{sc}$) determined for resistive circuits is replaced by their phasor representations, $V_{oc}$ and $I_{sc}$
- The Thévenin resistance, $R_{TH}$, is replaced by a Thévenin impedance, $Z_{TH}$.

Thus, the Thévenin and Norton equivalent circuits in the frequency domain are as shown in Figure 3.

![Thévenin and Norton equivalent circuits](image)

Figure 3. Thévenin and Norton equivalent circuits.

Since Thévenin’s and Norton’s Theorems both apply in the frequency domain, the approaches we used for source transformations in the time domain for resistive circuits translate directly to the frequency domain, with impedances substituted for resistances and phasors used for voltage and current terms.

In order to determine the load necessary to draw the maximum power from a Thévenin equivalent circuit, we must re-derive the maximum power result obtained previously for resistive circuits, substituting impedances for admittances and using phasors for source terms. We will not derive the governing relationship, but will simply state that, in order to transfer the maximum power to a load, the load impedance must be the complex conjugate of the Thévenin impedance of the circuit being loaded. Thus, if a Thévenin equivalent circuit has some impedance $Z_{TH}$ with a resistance $R_{TH}$ and a reactance $X_{TH}$, the load which will draw the maximum power from this circuit must have resistance $R_{TH}$ and a reactance $-X_{TH}$. The appropriate loaded circuit is shown in Figure 4 below.

![Load impedance to draw maximum power from a Thévenin circuit](image)

Figure 4. Load impedance to draw maximum power from a Thévenin circuit.

Example 8: Determine the Thévenin equivalent circuit seen by the load in the circuit below
In the circuit below, we have used the input frequency, \( \omega = 2 \text{ rad/sec} \), to convert the circuit to the frequency domain.

Removing the load and killing the source allows us to determine the Thévenin resistance of the circuit. The appropriate circuit is:

The parallel combination of two, 2\( \Omega \) resistors have an equivalent resistance of 1\( \Omega \). This impedance, in series with the \( j1\Omega \) impedance, results in a Thévenin impedance \( Z_{TH} = (1 + j1)\Omega \).

Replacing the source, but leaving the load terminals open-circuited, as shown in the figure below, allows us to determine the open-circuit voltage \( V_{OC} \).

Since there is no current through the inductor, due to the open-circuit condition, \( V_{OC} \) is determined from a simple resistive voltage divider formed by the two, 2\( \Omega \) resistors. Thus, the open-circuit voltage is:
\[ V_{oc} = \frac{2\Omega}{2\Omega + 2\Omega} \cdot 2\angle 0^\circ = 1\angle 0^\circ. \]

The resulting Thévenin equivalent circuit is shown below:

Example 9: Determine the Norton equivalent circuit of the circuit of example 8.

Since we determined the Thévenin equivalent circuit in Example 8, a source transformation can be used to determine the Norton equivalent circuit. Consistent with our previous source transformation rules, the short-circuit current, \( I_{SC} \), is equal to the open-circuit voltage divided by the Thévenin impedance:

\[ I_{SC} = \frac{V_{oc}}{Z_{TH}} = \frac{1\angle 0^\circ}{(1 + j1)\Omega} = \frac{1\angle 0^\circ}{\sqrt{2}\angle 45^\circ} = \frac{1}{\sqrt{2}} \angle -45^\circ \]

Since the impedance doesn't change during a source transformation, the Norton equivalent circuit is therefore as shown below:
Example 10: Determine the load impedance for the circuit of Example 8 which will provide the maximum amount of power to be delivered to the load. Provide a physical realization (a circuit) which will provide this impedance.

The maximum power is delivered to the load when the load impedance is the complex conjugate of the Thévenin impedance. Thus, the load impedance for maximum power transfer is:

\[ Z_L = (1 - j) \Omega \]

And the loaded Thévenin circuit is:

To implement this load, let us look at a parallel RC combination. With the frequency \( \omega = 2 \) rad/sec, the frequency domain load looks like:

Combining parallel impedances results in:

\[ Z_L = \frac{-j}{R + \frac{j}{2C}} = \frac{R}{4C^2} - \frac{jR^2}{2C} \Omega \]

Setting \( R = 2 \Omega \) and \( C = 0.25F \) makes \( Z_L = (1 - j) \Omega \), as desired, so the physical implementation of our load is as shown below: